



جامعة الناصر AL-NASSER UNIVERSITY

Graph Folding of Link Graph and Knot Graph

Dr.Mohammed Mahmoud Ali Redman Al-Shamiri

Assistant Professor of Algebra and Topological geometry

Department of Mathematics, Ibb University

AUTHORIZED BY AL-NASSER UNIVERSITY'S RESEARCH OFFICE

جميع حقوق النشر محفوظة لمكتب البحوث والنشر بجامعة الناصر

Dr.Mohammed Mahmoud Ali Redman Al-Shamiri

Assistant Professor of Algebra and Topological geometry

Department of Mathematics, Ibb University

In this article we introduced the definition of adjacency matrix folding and circuit adjacency matrix. We proved that a non-trivial graph folding of a link graph into itself is not a link graph and also we can fold any link graph by the graph folding. The sequences of graph foldings of a link graph with the different results are discussed. We proved that we can fold a knot graph by the graph folding under some conditions, by using the adjacency matrix folding and circuit adjacency matrix we described the graph folding.

Abstract

12

1.Introduction and background.

In mathematics a knot is a subset of 3-space that is homeomorphic to a unit circle. A link is a union of finitely many disjoint knots. The individual knots that make up a link are called its components, so a knot is a link with just one component[1,17].

A graph G consist of a set of elements called vertices together with a set of elements called edges ,the two sets having no common element. With each edge there are associated either one or two vertices called ends. An edge of G is loop or isthmus according as the number of its ends is 1 or 2. We restrict ourselves to finite graph, that is graphs for which V and E are both finite. A graph H is a subgraph of G if $V(H) \subseteq V(G)$ and each edge of H has the same ends in H as in G . The subgraph H of G is spanning subgraph of G if $V(H) = V(G)$. The subgraph of G for which $V(H) = W$ is a given subset W of $V(G)$ and $E(H)$ is the set of all edges of G having no end outside W , will be denoted by $G[W]$ [15,19].

A path in a graph G is a sequence of edges of the form e_1, e_2, \dots, e_n in which no edge is used more than once. We say that such a path goes from v_1 to v_n , we call v_1 the initial vertex of the path, and we call v_n the final vertex of the path. A closed path, or circuit, in a graph is a path whose initial and final vertices are the same. The degree of a vertex of a graph is the number of edges of the graph for which the vertex is an endpoint. A graph is called connected if for each pair of distinct vertices there is a path from one of them to other. If x and y are elements of $V(G)$ we say x and y are connected in G if there is a path from x to y in G . The relation of connection in G is an equivalence relation. Hence if $V(G)$ is non-null it can be partitioned into disjoint non-null subsets C_1, C_2, \dots, C_k such that two vertices of G are connected in G if and only if they belong to the same set C_i . The subgraphs $G[C_i]$ of G are the components of G . The graph G is connected if the number of its components is 1. A connected graph in which there is no circular path is a tree. To every link L there is a related the underlying graph, this graph is called link graph and satisfies the three properties:

1.is a finite connected graph.

2. is planer.

3. has the homogeneous vertex degree 4.

Every link graph with n vertices has $2n$ edges and $n+2$ countries[13].Every link graph is a knot projection on the plane, to obtain the types of link graphs for a knot of n crossing we look for the adjacency matrix. The adjacency matrix of the link graph is square matrix of size $2n \times 2n$ which is symmetric has integer values 0,1,2 and for every row and every column the sum is 4 and the main diagonal are zeros. For $n=1$ we have only one adjacency matrix also for $n=2$ we have only one adjacency matrix. For any n we get all possible adjacency matrices, and draw the link graph for all of them. Every link graph gives a decomposition of the plane in connected regions the so called countries. By coloring these countries

chessboard like manner with the two colors black and white such that the unbounded country is white. From the black countries (resp. white countries) one gets the corresponding graph of these black countries (resp. white countries), which called knot graph. Let G be a given graph and e any arbitrary one of its edges [1,17].

The Tutte polynomial of G is a polynomial in two variables which satisfies the following properties:

- (1) for G with n vertices and m edges
- (2) if e is a bridge.
- (3) if e is a loop.
- (4) where $G - e$ (respectively G / e) denotes the graph obtained from G by deleting (resp. contracting) e .
- (5) if e has at least two edges and e is a bridge (resp. loop) of G , $T(G, x, y) = T(G - e, x, y) + T(G / e, x, y)$ (resp. $T(G, x, y) = T(G - e, x, y) + T(G / e, x, y)$).

Note that properties (1),..., (5) allow the computation of $T(G, x, y)$ for any graph.

In 1977 Robertson [18] introduced this definition A map F , where M and N are Riemannian manifolds (see [14]) of dimension m, n , respectively, is said to be an isometric folding of M into N , if and only if for any geodesic path γ the induced path $F(\gamma)$ is a piecewise geodesic and of the same length as γ . If F does not preserve length, then F is a topological folding. and the folding of manifold into another, or into itself are studied by El-Kholy [11,12] and El-Ghoul [2-6]. El-Kholy introduced the definition of graph folding. Let G and H be graphs and f be a continuous function. Then f is called a graph map, if

- i) for each vertex v is a vertex in H
- ii) For each edge e in G , $f(e)$ is an edge in H .

We call a graph map f a graph folding if and only if f maps vertices to vertices and edges to edges, i.e., for each v and for each e . Else that, we can not get a graph folding [12]. This article is continuation to [7,8] and we introduced in it the graph folding of link graph and knot graph and the related that for the adjacency matrix [12].

Theorem 1.1.

Let G be a complete graph, then there is no non-trivial graph folding can be defined for G .

Proof. See [12].

Proposition 1.1.

Any tree T can be folded into itself by a sequence of graph foldings onto an edge.

Proof. See [12].

2. Main results.

Proposition 2.1 .

The graph folding image of any link graph is not a link graph.

Proof.

Let L be a link graph and f be a graph folding from L into itself. Then f must be folded at least one vertex on another and hence one edge is folded on another or f folded at least one edge on another. In both cases the homogenous vertex degree four is not found. So that $f(L)$ is not link graph.

Definition 2.1.

A trivial link graph is a link graph, the number of its countries which bounded by two edges equal the number of its vertices. and a composite link graph is a link graph can be partitioned into two or more link graphs. else is called a prime link graph

Theorem 2.1.

Any link graph L , can be folded by graph folding.

Proof.

Let L be a link graph of n vertices, the number of its edges is $2n$ and the number of countries. Then there is at least one multiple edge. Hence we can define a graph folding for it or else every country is bounded by at least 3 edges and there is at least two vertices, can be folded

Theorem2.2.

Any prime link graph of n vertices, can be folded by sequence of graph foldings its end is a complete graph.

Proof.

Let L be a prime link graph of n vertices from theorem 2.1, there are a graph folding from L into itself .

is a graph but not link graph its incidence matrix can be partitioned (to details see[12]) and hence a graph folding can be defined until we get a graph with no multiple edge. Thus every country is bounded by at least 3 edges, by graph folding again we get is a complete graph and hence can not folded after that (theorem1.1).

Proposition 2.2

Every Link graph with 5 vertices or less, has at least one multiple edge .

Proof.

If there is no multiple edge then every country is bounded by at least 3 edges. Thus the number q of pairs (e, c) of an edge e and an incident country c is at least $3(n + 2)$. Because every edge e belongs to 2 countries we have $q = 2k$. This gives i.e .

Therefore we have found for or less every link graph has at least 1 multiple edge.

Corollary 2.1.

The graph folding image of a prime link graph of 4 vertices is a complete graph of degree 4.

Proof . The proof is clear.

Now we define folding of adjacency matrix by:

Definition2.1 .

Let A be an adjacency matrix and f be a map from A into itself defined by , then f is called adjacency matrix folding.

Example2.1.

Let A be an adjacency matrix of link graph in example2.2 and f be an adjacency matrix folding from A into itself as define below :

Note that is a matrix satisfies three conditions:

1. is square matrix of size
2. is symmetric has integer values 0 and 1
3. For every row and every column the sum is 2 and the main diagonal are zeros.

Definition2.2 .

The matrix which satisfies the three conditions above is called circuit adjacency matrix.

Theorem2.3.

A trivial link graph L of even vertices can be folded by sequence of graph foldings into an edge if and only if the adjacency matrix folding is a circuit adjacency matrix of size.

Proof.

Let L be a link graph and A its adjacency matrix of size , is even and is a circuit adjacency matrix, by one fold by graph folding of the graph which represented by ,we

get a tree graph and by Proposition 1.1 the proof is complete. The second direction, the proof come directory by make a tree and obtained circuit graph of n vertices.

Corollary 2.2 .

A trivial link graph L of odd vertices with adjacency matrix can be folded by sequence of graph foldings into a complete graph if and only of the adjacency matrix folding is a circuit adjacency matrix of size.

Proof. The proof is clear.

Rmark2.1.

If the link graph is a composite (can be partitioned into two link graphs) ,then the above theorems is not necessary hold.

Example2.2.

Let L be a link graph of 6 vertices and A be an adjacency matrix, we can be partitioned into two link graphs as shown below Fig.(1):

Fig.(1)

We can be folded L by graph folding into edge see Fig.(2), but is not a circuit adjacency matrix .

Fig.(2)

Proposition2.2

Let is a graph folding of knot graph into itself then not necessary

By the following example we illustrate the proposition above.

Example2.3

Let be a link graph of 6 vertices represents a knot of 6 crossing and be a knot graph obtained from see Fig.(3). We observe that is a complete graph and by theorem1.1 we can not define a non trivial graph folding for .

Fig.(3)

Remark2.3

From the example above the adjacency matrix folding is a trivial

Lemma2.1

Every knot graph of a knot of n crossing ,obtained from link graph L of which has at least one multiple edge is circuit graph or has at least one multiple edge.

Theorem 2.4

Let K be a knot graph of a knot of n crossing, then a non trivial graph folding can be defined for K if the adjacency matrix folding of a link graph L of K is not trivial.

Proof.

Let A be an adjacency matrix of a link graph L , since K is non trivial, then L has at least one multiple edge and hence the region shaded black more than the region shaded white so that the knot graph has at least one multiple edge or is circuit graph and graph folding can be defined for it.

Theorem 2.5

Let K be a knot graph of a knot of n crossing, obtained from prime link graph L of which has at least one multiple edge, then K folded into an edge.

Proof The proof is clear.

Remark 2.3

In the special case the knot graph K which obtained from prime link graph L of 4 vertices, folded into a complete graph.

REFERENCES

- (1) C.Adams: The knot Book, Freeman ,San Francisco, (1994).
- (2) M. El-Ghoul, The deformation retraction and topological folding of manifold, comm..Fac.Sic. Univ. Ankara series Av.37-pp(1998) (1-7).
- (3) M .El-Ghoul, The deformation retract of complex projective and its topological folding, Journal of material science (30), England (1995) 4145-4148.
- (4) M. El-Ghoul, Folding of manifolds, Ph.D. Thesis Univ. Tanta. Egypt ,(1985).
- (5) M. El-Ghoul & H.I. Attiya, The Dynamical Fuzzy topological space and its folding the Journal of Fuzzy mathematics Vol.12 ,No.3, 2004.
- (6) M. El-Ghoul, Fractionl folding of manifold.Chaos,Soliton Fract., England;12 (2002) 1019 -1023.
- (7) M. El-Ghoul, A.I.Elrokh & M. M .Al-Shamiri, Folding of trefoil knot and its graph,Journal of Mathematics and Statistics2(2)368-372,2006.
- (8) M. El-Ghoul, A.I.Elrokh & M. M .Al-Shamiri, Folding of tubular trefoil knot

- International Journal of applied Mathematics,(2007).
- (9) M. El-Ghoul, & M. M .Al-Shamiri: Retraction of braid and braid group, Asian Journal of algebra,1 (1):1-9 (2008).
- (10) M. El-Ghoul, & M. M .Al-Shamiri: Some types of dynamical 3-dimensional braid, international journal of geometrical methods in modern physics,Vol.5, No5, (2008) 755-764.
- (11) E. El-Kholy ,Isometric and topological folding of manifolds. Ph.D Theses, University of Souththampton,UK,1977.
- (12) E. El-Kholy and A.El-Esawy,Graph Folding of Some Special Graphs, Journal of Mathematics and Statistics1 (1) 66-70,2005
- (13) A. I. Elrokh, Graphs,Knots and Links, Ph.D. Thesis. Univ.Ernst-Moritz-Arndt Greifswad.(2004).
- (14) W.S.Massay,Algebraic Topology.An Introduction, Harcout,Brace and World, New York ,1967.
- (15) W.Mayeda: Graph theory. Jon Wiley & Sons, Inc ,Canada(1972).
- (16) J.R. Munkers, Topology,A First Course, Prentice –Hall Inc., Englewood Cliffs ,New Jersey,U.S.A,(1975).
- (17) K .Murasugi ,Knot Theory and its Applications , Birkhaser, Boston, MA , 1996.
- (18) S.A.Robertson, Isometric folding of Riemannian manifold,ProcRoy Soc. Edinburgh 77(1977) 275-284.
- (19) R.J.Wilson: Introduction to graph theory. Oliver & Boyed, Edinburgh, (1972).